

Assignment 2: Basic Set Operations

Due: December 12, 2019

Instructions

Make sure to submit your homework on **SINGLE-SIDED 8.5×11 INCH PAPER**. Write your **NAME ON EVERY PAGE** and **DO NOT STAPLE OR FOLD** the sheets.

In this and future homeworks, questions marked as optional will not be used for quizzes or final exam questions.

1. Find a counterexample to the statement, “All odd natural numbers are prime.” What is the least counterexample (NOTE: 1 is **NOT** prime)?
2. Here are some standard definitions of set operations. I’m using the symbol “:=” to mean “equals by definition.” For any sets A and B ,

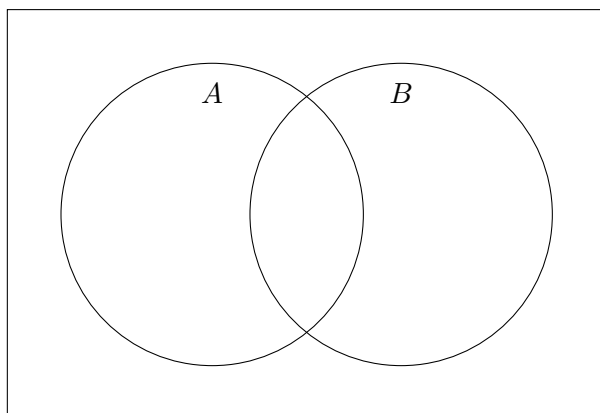
Union: $A \cup B := \{x : x \in A \vee x \in B\}$.

Intersection: $A \cap B := \{x : x \in A \wedge x \in B\}$.

Difference (Relative Complement): $A - B := \{x : x \in A \wedge x \notin B\}$.

Symmetric Difference: $A \triangle B = (A - B) \cup (B - A)$.

For each of the four operations above, redraw the Venn diagram below, shading the region corresponding to the result of the operation.



3. Illustrate via a venn diagram the following identity for any sets A and B .

$$A \triangle B = (A \cup B) - (A \cap B).$$

4. Prove the identity in the last problem directly for any sets A and B .
5. Let A and B be any sets. Which of the following sets must be empty? For each of those that need not be, give a specific, concrete example where it is not (your examples should be as small as possible).

- (a) $A \cup \emptyset$
- (b) $A \cap \emptyset$
- (c) $A - \emptyset$
- (d) $A \times \emptyset$
- (e) $A - B$
- (f) $A - A$
- (g) $(A - B) \cap (B - A)$
- (h) $(A - B) - (B - A)$
- (i) $A - (B - A)$
- (j) $A - (A - B)$
- (k) $A - (A - A)$
- (l) $(A - B) \cap B$